An Analytical Expression for k-connectivity of Wireless Ad Hoc Networks

Nagesh K. N*1, Satyanarayana D2, and M.N Giri Prasad3
1 Middle East College, Knowledge Oasis, Al Rusyal, Sultanate of Oman
2 ECE Department, RGM CET, Nandyal- 528502, A.P, India
3 ECE Department, JNTUCE, Anantapur- 515002, A.P, India
*Corresponding author, e-mail: nagesh@mec.edu.om

Abstract

Over the last few years coverage and connectivity of wireless ad hoc networks have fascinated considerable attention. The presented paper analyses and investigates the issues of k-connectivity probability and its robustness in wireless ad hoc network while considering fading techniques like lognormal fading, Rayleigh fading, and nakagami fading in the ad hoc communication environment, by means of shadowing and fading phenomenon. In case of k-connected wireless sensor network (WSNs), this technique permits the routing of data packets or messages via individual (one or more) of minimum k node disjoint communication paths, but the other remaining paths can also be used. The major contribution of the paper is mathematical expressions for k-connectivity probability.

Keywords: Log Normal Fading, Rayleigh Fading, Nakagami Fading, K-connectivity, Ad hoc network

1. Introduction

An Ad hoc Network is a kind of multi-hop wireless network formed by mobile nodes which are capable of communicating with each other without an infrastructure. This spontaneous network is adaptable to the extremely dynamic topology resulted from the quality of network nodes and therefore the dynamical propagation conditions. Associate degree Ad hoc network may be an assortment of wireless mobile nodes dynamically forming a short lived network while not the employment of any preexistent network infrastructure or centralized administration. Ad hoc network is a type of decentralized wireless network, and it is referred as ad-hoc because each node is enthusiastic to forward its information or data for other nodes, and so the fortitude of which nodes forward data is made vigorously based on the network connectivity. This is the matter of fact that the wireless communication medium do suffers from multiple propagation impairments like path loss, due to multi-path fading and shadowing effects [4]. The reference [5] explores and hence comes out with a system modeling mathematical expression for outage likelihood while taking into account of the fading techniques like Rayleigh fading and shadowing effects. The probability of node isolation or network isolation and thus resulting probability of network connectivity or coverage by taking into account of effects like shadowing and fading has to consider and analyze the randomness in network channel with the help of channel diversity.

In successive literature reviews; a number of studies have been performed and most of the literatures have been studied for investigation of individual connectivity issue (1-connectivity) in multi-hop communication networks. In reference work [6] the connectivity problem in a lognormal shadowing environment has been studied and analyzed and the presented contribution provides tight lower bound of the minimum nodes density for achieving an approximately unquestionably network connectivity. The reference [7] proposes and successfully develops the network connectivity approach by taking into account of effects, shadowing as well as fading simultaneously. Then while these all existing systems or approaches consider single connectivity (1-connectivity) approach and it fact neither of these techniques considers and analyzes k-connectivity (k > 1) scheme in system development and realization. In order to optimize the consistency in communication and to ensure the QoS requirements, in major cases, it becomes necessity to satisfy the needs and requirement of k-connectivity (k ≥ 1). In this subscript, the proposed k-connectivity approach refers towards network characteristics where a randomly assessed node poses minimum of k neighboring nodes. Generally, the comprising nodes or network transceiver terminals are spread on the
basis of a Poisson point process (PPP). One the nodes are distribute according to PPP, then the isolation probability or connectivity for a node is computed with a belief that there are at least k-neighbouring nodes. Accomplishing above mentioned phases or steps k-connectivity probability computed for the considered network. Reference [8] presents the comprehensive result obtained. Moreover, to hold the normally distributed point the characteristics for k=1 is presented in reference [9]. In reference [10] the distribution of threshold range in asymptotic approach for k-connectivity has been done, while taking into account that it is more than one degree parameter (k>1), for points lying inside a geometrics (circular or a square) are distributed in homogeneous way. The results obtained while considering the approximate value of k-connectivity for fixed network by employing least value of degree k, has been employed and analyzed in reference [11]. The study and analysis of k-connectivity with the presence of channel randomness has been done in reference [12].

The analysis of least node degree and the k-connectivity probability with the consideration of simple geometric model which itself is deterministic and is dependent on distance has been done in paper [13]; likewise wireless channel can be more realistic in functional manner. Significantly the randomness introduced by shadowing effect that is caused by obstacles must be considered.

Since the emergence of wireless communication network and especially in Ad hoc network, because of its dynamicity and robustness, a number of researches have been conducted and systems were proposed. For ensuring the higher quality of service of network by employing k-connectivity approach in WSNs, in [14] a noble approach with random graph model or “cryptography approach ” was derived that ultimately estimates the asymptotic probability whether a node in WSN is connected by means of k-connectivity in secured way. The goal of this work was to deliver a secured k-connectivity approach. Therefore, in order to accomplish this goal a fault tolerant k-connectivity approach was realized in [15]. For accomplishing the objective of a fault tolerant k-connectivity approach a localized fault-tolerant topology control algorithm can be developed that effectively control and constructs fault-tolerant networks by means of scheming transmit energy and faction of mobile nodes. The network maintenance approach in dynamic k-connectivity was studied in [16] where minimum k-edge/vertex connected network components of a grid accomplishing the repeated dynamic updates were realized in implementation. Reference [17] developed a mathematical approach for estimation of kappa-connectivity that finds out the maximum node density in realization area and thus it gave a sound mechanism for connectivity assurance. But the further enhancement for bridging the robustness gap in connectivity issue was done by [18] in cognitive radio network. A noble approach called Distributed k-connectivity maintenance was developed in [19] that deliver high end maintenance of k-connected network. In [20] K-Connectivity was realized with Shadowing and Nakagami Fading Wireless Multi-Hop Networks. This work delivers the result that could be effectively used in real time systems. A three dimensional network for realizing the network connectivity can be done by designing a combination of various patterns made for whole coverage and 2 agent connectivity approach [21].

The dominant as well as significant properties of wireless Ad hoc networks are minimum node degree and k-connectivity probability. The analysis of characteristics while taking into account of fading techniques like Rayleigh fading, Lognormal shadowing channel, Nakagami fading and superimposed lognormal shadowing has been presented in this paper.

The primary assumptions and model are provided in section 2, and 3 represents the analytical evaluation of k-connectivity probability. The results obtained in the form of numerical and simulation results have been presented and discussed subsequent in section 4 which is followed by the conclusion section mentioned in section 5 of this manuscript.

2. System Model

According to the uniform Poisson point process, it is considered that all the comprising nodes in network are distributed randomly. Consider the number of consisting nodes available or functional in per square unit (0 < \lambda < \infty) is presented by \lambda and the two dimensional (2 − D) Poisson point process over R_2 as stated by variable N. The respective position of the encompassing nodes is represented by the points of the process.

In case of provided definite subset \Lambda \in R_2 of size\nu(\Lambda), N(\Lambda) represents the number of consisting nodes in \Lambda and this variable states the Poisson random variable with \lambda\nu(\Lambda) intensity.
Since, in case of k-connectivity approach a particular node is often or normally connected with k number of nodes, therefore the resulting or general expression for k-connectivity probability can be presented by mathematical expression, mentioned below [13].

\[ P(d_{\min} \geq k) = \left( 1 - \sum_{n=0}^{K} \frac{(\lambda \pi d_0^n)^N}{n!} e^{-\lambda \pi d_0^2} \right)^n \]  

(1)

In the presented expression, the distance vector or parameter \(d_{\min}\) is a node indiscriminately chosen that poses minimum number of neighbor \(k\). The variable \(n\) refers the total number of comprised nodes in operational network.

3. Analytical K-Connectivity Probability Estimation

The proposed K-connectivity probability for wireless Ad hoc network has been estimated in this section. The probability estimation has been done in the presence of different fading environments like shadow fading, Rayleigh fading and Nakagami fading encompassed with superimposed lognormal shadowing.

3.1. Lognormal Shadowing and Small Scale Fading

This section discusses the K-connectivity probability with minimum node degree and in the communication environment with lognormal shadow fading channel

The channel attenuation caused due to Shadow effect, that is \(10^{\frac{-z_{ij}}{10}}\) is in general distributed in lognormal manner amid two nodes \(i\) and \(j\) and it is same without emphasizing which is the transmitting node and which is the receiving node in communication. Therefore, \(Z_{ij} \sim N(0, \sigma^2)\) for all \(i, j\) where the parameter \(\sigma\) is equal or similar for all nodes \(i\) and \(j\).

Comprising nodes \(i\) and \(j\) poses a communicative correlation if received power increases from the predefined or provided threshold \(P_{\min}\), i.e.,

\[ K \frac{P_r}{r^2} 10^{\frac{-z_{ij}}{10}} > P_{\min} \iff r < \frac{h_{Z_{ij}}}{\sigma}, h = \frac{ln10}{10} \]  

(2)

In above mentioned expression the variable \(\alpha\) refers the distance-loss exponent parameter.

The shadow fades in between defined nodes \(i\) and \(j\) given by expression, \(10^{\frac{-z_{ij}}{10}}, r\) represents the distance between the nodes, and \(K\) refers a constant variable. The probability density function (PDF) of the distance \(r\) between a transmitter-receiver pair is given in the form of the following expression:

\[ f_r(r) = \begin{cases} \frac{2r}{R^2}, & 0 \leq r \leq R \\ 0, & \text{otherwise} \end{cases} \]  

(3)

Probability that there might be any direct connection between these nodes is given in expression [22]

\[ P(r_0) = P\{R < r_n \exp\left(\frac{h_{Z_{ij}}}{\sigma}\right)\} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} \int_{r_0}^{r_0 \exp\left(\frac{h_{Z_{ij}}}{\sigma}\right)} \frac{2r}{R^2} dr \, dz \]  

(4)

Where, \(r_n = \left(\frac{P_{\min}K}{P_{\min}^{\alpha}}\right)^{\frac{1}{n}}\)

Solving the mathematical expression (4) we get:

\[ P(r_0) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} \left[ r_0^{-2}\exp\left(\frac{h_{Z_{ij}}}{\sigma}\right) \right] \frac{dz}{z} \]  

An Analytical Expression for k-connectivity of Wireless Ad Hoc Networks (Nagesh K. N.)
Further simplification can be done by considering equation [5] and it can be given as

\[ \int_{-\infty}^{\infty} e^{-\left(\frac{r_0^2}{2}\right) + \frac{ub}{2}} dx = e^{\frac{ub}{2^2}} \]  

(6)

Combining equation (5) and (6) we can get

\[ P \left( r_0 = \frac{r_0^2}{r_0^2} \right) \exp \left( \frac{2b^2\sigma^2}{a^2} \right) = \frac{u}{r_0^2} \]

(7)

Where \( u = r_0^2 \exp \left( \frac{2b^2\sigma^2}{a^2} \right) \)

Consider, the number of nodes are given by a random variable that represents the number of comprising nodes through which the node present at zero position or at (0, 0) has a direct connection. In this case, the probability mass function of variable \( N_n \) can be written as follows [20]:

\[ P (N_n = n) = \lim_{r_0 \to \infty} e^{-\lambda \pi r_0^2} \sum_{k=n}^{\infty} \left( \frac{\lambda \pi r_0^2}{k^2} \right)^n \left( \frac{n}{n} - \frac{u}{r_0^2} \right)^k \]

(8)

Considering \( m = k - n \) in the above mentioned mathematical expression (8), the simplification can be made as follows:

\[ P (N_n = n) = \lim_{r_0 \to \infty} e^{-\lambda \pi r_0^2} \sum_{m=0}^{\infty} \left( \frac{\lambda \pi r_0^2}{m+n} \right)^m \left( \frac{n}{n} - \frac{u}{r_0^2} \right) \left( 1 - \frac{u}{r_0^2} \right) = \frac{(\lambda \pi u)^n}{n!} e^{-\lambda \pi u} \]

(9)

Combining the equations (1) and (9) the K-connectivity probability can be calculated and it can be given as [13]:

\[ P(d_{min} \geq k) = \left( 1 - \sum_{N=0}^{K-1} \frac{(\lambda \pi r_0^2 \exp \left( \frac{2b^2\sigma^2}{a^2} \right))^N}{N!} e^{-\lambda \pi r_0^2 \exp \left( \frac{2b^2\sigma^2}{a^2} \right)} \right)^n \]

(10)

3.2. Rayleigh with Superimposed Lognormal Shadowing Channel

The analysis of k-connectivity with consideration of extensive shadowing effects and small scale Rayleigh fading is done in this presented section. Signal to Noise Ratio (SNR) for Rayleigh fading’s probability distribution function has been given by [21].

\[ f_y(a) = \frac{1}{y} e^{-\frac{a}{y}} \]

\( y \) represents is the average SNR, and received immediate SNR. The success probability can be defined by the following expression:

\[ P_s(y) = \int_{a}^{\infty} e^{-\frac{a}{y}} da = e^{-\frac{y}{y}} \]

(11)

Where \( y = \frac{k P_\text{Rx}r^-a}{W} \)
The connectivity probability whether the nodes are directly connected or not can be obtained in the same way as done in descending section, given as

\[ P \left\{ R < r_n e^{\frac{hr}{\alpha}} \right\} = P(r_0) = \int_{0}^{\infty} \frac{1}{2\pi r_0} e^{-\frac{z^2}{2r_0^2}} \psi \cdots \int_{0}^{\min[r_0, r_n e^{\frac{hr}{\alpha}}]} \frac{2r}{r_0^2} \ dr \ dz \]  

(12)

The above presented expression can be further simplified and can be given as:

\[ P(r_0) = \int_{0}^{\infty} \frac{1}{2\pi r_0} e^{-\frac{z^2}{2r_0^2}} \frac{2r}{r_0^2} e^{-\frac{y}{r_0^2}} \ dr \]  

(13)

The mathematical expression can be obtained by solving the integration function is given as:

\[ l_1 = \int_{0}^{r_n \exp \left( \frac{hr}{\alpha} \right)} r \exp(-\beta r) \ dr \]  

\[ \beta = \frac{W \Phi}{K_{Ptx}} \]  

(14)

We have,

\[ \int_{0}^{u} (x - a) \exp(-\beta(x - b)n) \ dx = \frac{\int_{a}^{u} (x - b) \exp(-\beta x n) \ dx}{n \beta n} - \frac{(a - b) \int_{a}^{u} (x - b) \exp(-\beta x n) \ dx}{n \beta n} \]  

(15)

Consider \( a = b, b = 0, n = \alpha, u = r_n \exp \left( \frac{hr}{\alpha} \right) \)

Extracting and considering expressions (15) and (14) it can be obtained as

\[ \int_{0}^{r_n \exp \left( \frac{hr}{\alpha} \right)} r \exp(-\beta r) \ dr = \frac{\sum_{k_1=0}^{\infty} \left( \frac{W \Phi}{K_{Ptx}} \right)^{\left( \frac{2}{\alpha} + K_1 \right)} r_n \left( \frac{2}{\alpha} + K_1 \right) \exp \left( \frac{hr \left( \frac{2}{\alpha} + K_1 \right)}{r_n} \right)}{(\frac{2}{\alpha} + K_1) K_1 ! (a \left( \frac{2}{\alpha} + K_1 \right))} \]  

(16)

Considering two equations (16) and (13), \( P(r_0) \) can be obtained and it can be given as follows

\[ P(r_0) = \int_{0}^{\infty} \frac{1}{2\pi r_0} e^{-\frac{z^2}{2r_0^2}} \frac{2r}{r_0^2} \sum_{k_1=0}^{\infty} \left( \frac{W \Phi}{K_{Ptx}} \right)^{\left( \frac{2}{\alpha} + K_1 \right)} r_n \left( \frac{2}{\alpha} + K_1 \right) \exp \left( \frac{hr \left( \frac{2}{\alpha} + K_1 \right)}{r_n} \right) = \]  

\[ \frac{2r}{r_0^2} \left( \frac{W \Phi}{K_{Ptx}} \right)^{\left( \frac{2}{\alpha} + K_1 \right)} r_n \left( \frac{2}{\alpha} + K_1 \right) \sum_{k_1=0}^{\infty} \left( \frac{hr \left( \frac{2}{\alpha} + K_1 \right)}{r_n} \right)^{(\frac{2}{\alpha} + K_1) K_1} \exp \left( -\frac{hr \left( \frac{2}{\alpha} + K_1 \right)^2}{2} \right) = \frac{u}{r_0^2} \]  

(17)

From equation (17), (9) and (1) the parametric or performance comparison of k-connectivity probability will be done [25]

\[ P(d_{min} \geq k) = \left\{ 1 - \sum_{n=0}^{K-1} \left( \frac{\lambda n u}{N!} e^{-\lambda n u} \right) \right\}^n \]

Where

\[ u = \frac{2}{a \left( \frac{2}{\alpha} + K_1 \right) K_1 ! (a \left( \frac{2}{\alpha} + K_1 \right))} \sum_{k_1=0}^{\infty} \left( \frac{hr \left( \frac{2}{\alpha} + K_1 \right)}{r_n} \right)^{(\frac{2}{\alpha} + K_1) K_1} \exp \left( -\frac{hr \left( \frac{2}{\alpha} + K_1 \right)^2}{2} \right) \]  

(18)

### 3.3. Nakagami-m Fading with Superimposed Lognormal Shadowing

This section discusses the derivation of mathematical expression for the k-connectivity of Nakagami-m fading channel while considering superimposed lognormal shadowing effect.
The fact that the nodes are directly connected or not and their probability of direct connection is computed in the same way as done in previous section.

\[
P(r_0) = \int_{-\infty}^{\infty} f_x(z)dz \cdot P_x(\psi) \int_0^{r_0} \exp \left( \frac{hr}{\alpha} \right) \frac{2r}{r_0^2} dr
\]  \hspace{1cm} (19)

The above given expression \( f_x(z) \) refers to the shadow fading probability distribution function given by [14].

\[
f_x(z) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi \sigma}} e^{-\frac{z^2}{2\sigma^2}}
\]  \hspace{1cm} (20)

\( p_x(\psi) \) is denoted as the probability at which SNR received at receiver point is greater than the threshold SNR [16].

The probability distribution function of Nakagami-m fading SNR is given as:

\[
f_\psi(y) = \left( \frac{m}{y} \right)^m \frac{1}{\Gamma(m)} y^{m-1} e^{-\frac{my}{y}}
\]  \hspace{1cm} (21)

\[
P_\psi(\psi) = pr\{y \geq \psi\} = \int_\psi^{\infty} f_\psi(y)dy
\]

\[
P_\psi(\psi) = e^{-\frac{m\psi^\alpha}{K_\phi} \sum_{k=0}^{m-1} \left( \frac{m\psi^\alpha}{K_\phi} \right)^k}
\]  \hspace{1cm} (22)

If we put equation (20) and equation (22) in equation (19) we will get:

\[
P(r_0) = \int_{-\infty}^{\infty} f_x(z)dz \cdot \int_0^{r_0} \exp \left( \frac{hr}{\alpha} \right) \frac{2r}{r_0^2} \cdot \frac{m\psi^\alpha}{K_\phi} \sum_{i=0}^{m-1} \frac{\left( \frac{m\psi^\alpha}{K_\phi} \right)^i}{i!} \cdot \frac{1}{r_0^2} dr
\]  \hspace{1cm} (23)

\[
P(r_0) = \left( \frac{r_0^2}{2} \right)^{-m} \sum_{i=0}^{m-1} \frac{\left( \frac{m\psi^\alpha}{K_\phi} \right)^i}{i!} \left( \frac{r_0^2}{2} \right)^{m-1-i} \sum_{k=0}^{\infty} \left( -1 \right)^k \frac{\left( \frac{m\psi^\alpha}{K_\phi} \right)^k}{k!} \exp \left[ \frac{\left( \frac{r_0^2}{2} \right)^{m-1-i}}{\left( \frac{m\psi^\alpha}{K_\phi} \right)^i} \right] \exp \left[ \frac{\left( \frac{r_0^2}{2} \right)^{m-1-i}}{\left( \frac{m\psi^\alpha}{K_\phi} \right)^i} \right]
\]  \hspace{1cm} (24)

\[
u = \left( \frac{r_0^2}{2} \right)^{-m} \sum_{i=0}^{m-1} \frac{\left( \frac{m\psi^\alpha}{K_\phi} \right)^i}{i!} \left( \frac{r_0^2}{2} \right)^{m-1-i} \sum_{k=0}^{\infty} \left( -1 \right)^k \frac{\left( \frac{m\psi^\alpha}{K_\phi} \right)^k}{k!} \exp \left[ \frac{\left( \frac{r_0^2}{2} \right)^{m-1-i}}{\left( \frac{m\psi^\alpha}{K_\phi} \right)^i} \right] \exp \left[ \frac{\left( \frac{r_0^2}{2} \right)^{m-1-i}}{\left( \frac{m\psi^\alpha}{K_\phi} \right)^i} \right]
\]  \hspace{1cm} (25)

Therefore, the probability of \( k \)-connectivity is obtained and then presented by the equations (9), (25) and (1) as

\[
P(d_{min} \geq k) = \left( 1 - \sum_{H=0}^{K-1} \left( \frac{\lambda \psi u}{K} \right)^H e^{-\lambda \psi u} \right)^n
\]  \hspace{1cm} (26)

4. Numerical and Simulation Results

An analytical model has been developed using MATLAB and the simulation framework and numerical results are attained from them keeping into consideration that the constraints are selected as \( k = 10dB, P_{tx} = 1mW, W = 0.01mW, \psi = 10dB \). After that we properly choose the other constraints which are \( m, \lambda, \alpha, \) and \( \sigma \). A constant dimension of 100mX100m has been used for the simulation purpose. Randomly distributed nodes are used to get distributed over the simulation framework using the Poisson process and with respect to the channel model diverse links are established. Finally after the average 1000 simulation framework results the \( k \)-connectivity probability is calculated. Figure 1 and 2 for various channel conditions represents the simulated network topologies. In Figure 3 the change in \( k \)-connectivity probability with respect to node density \( (\lambda) \) for diverse values of sigma in logarithmic shadowing channel can be visualized.
An Analytical Expression for $k$-connectivity of Wireless Ad Hoc Networks (Nagesh K. N.)

Figure 1. Simulated network topology (sigma=2)

Figure 2. Simulated network topology (sigma=4)

Figure 3. $K$-Connectivity Probability V/S Node Density Lambda (With Respect to Lognormal Shadow Fading)
From the results we come to know that $\lambda$ and $\sum$ are directly proportional to the $k$-connectivity probability. In order to accomplish $k$-connectivity the minimum node density which is required, can be determined from the result. The variation between logarithmic standard deviation ($\sum$) and $k$-connectivity probability is presented in Figure 4 with respect to the path loss exponent ($\alpha$). $\alpha$ as expected is inversely proportional to the $k$-connectivity and $\sigma$ is directly proportional to it ($\alpha$).

The variation between node density ($\lambda$) and $k$-connectivity probability is presented in Figure 5 with different values of $\sigma$ according to the Rayleigh fading and superimposed lognormal fading. From the result it can be found that the $k$-connectivity probability is directly proportional to node density and $\sigma$. The illustration of $k$-connectivity with respect to $\lambda$ (node density) and the different values of $\sigma$ have been kept into consideration in the Nakagami fading channel when the lognormal shadowing has been superimposed and has been shown in Figure 6. It can be clearly realized from the result that $k$-connectivity probability is directly proportional to $\lambda$ and $\sigma$. It can also be calculated from the result to find what is the minimum $\lambda$ required to achieve the $k$-connectivity.

![Figure 4. K-Connectivity Probability V/S Standard Deviation Sigma (With Respect to Lognormal Shadow Fading)](image1)

![Figure 5. K-Connectivity Probability V/S Lambda (With Respect to Rayleigh fading)](image2)
5. Conclusions

The study and investigation of $k$-connectivity in the wireless multi-hop network with the distribution of node has been done uniformly in the paper. In order to support the theoretical background and mathematical expression has been derived for providing practical result with the help of theoretical background. The requirement for achieving more than one connected network has been obtained in the paper. To cover the certain region using $k$-connectivity probability network it can easily be estimated the number of nodes that are required from the paper. The results which are achieved work as practical values for the development and research work and the simulation and designing work for the ad hoc network take place according to it. In real ad hoc system the results can also be implemented. In various multi-hop networks the results obtained from the paper can be applied and for the Nakagami-m fading channel and MRC diversity scheme the mathematical expression which are proposed in the paper can be used. MIMO (Multiple-Input and Multiple-Output) scheme might be the effective extension and future work for the proposed system.

References


